Resummed perturbative series of scalar quantum field theories in two-particle-irreducible formalism

Thesis points of a PhD thesis

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In my thesis I investigated the two-particle-irreducible (2PI) formalism in quantum field theory and its several applications, with particular emphasis on the renormalizability. In 2PI formalism physical quantities can be obtained as resulting from a partial infinite resummation of their perturbative series. In this sense it goes beyond the ordinary perturbation theory. For the field expectation values and the propagators one derives self-consistent equations, and it is a non-trivial question, how perturbative order by order renormalizability remains valid for the partially resummed perturbative series realized by the solution of the self-consistent equations.

The formalism was developed in the 1960s for non-relativistic field theories in condensed matter physics [6], and was generalized to relativistic theories in the 1970s [7]. It became an active research topic again in the last 10 years in the framework of non-equilibrium simulations. The renewed interest is due to the fact, that 2PI formalism is the only known solution to the so-called secular time-evolution problem. Diverging temporal evolution, which always appear in perturbative treatments of non-equilibrium evolution is avoided in 2PI formalism, and the simulated systems do thermalise. This feature can not be obtained in simulations based on ordinary perturbation theory.

My thesis deals with quantum field theories at zero temperature and *in equilibrium*. Its goal is to map and solve same problems of renormalization both from an analytic and a numerical point of view, together with applications of the newly developed techniques to specific scalar quantum field theories. It is important to stress that publications providing highly accurate numerical realization of 2PI renormalization in details are virtually missing from the literature. One of the aims of my research was to fill this gap.

The greater part of the examined models are used as effective theories of strong interactions. With my PhD thesis I wish to suggest new and improved treatments of models of phenomenological importance beyond ordinary perturbation theory, which is crucial when large coupling constants appear. It is not the task of this thesis to do phenomenology, it just proposes new adequate methods for further investigations in this direction.

In the analytic derivation of self-consistent renormalized equations the knowledge of the general theory of renormalization and functional techniques of quantum field theory were indispensable. For the study of models based on the orthogonal O(N) and the unitary U(N) groups, I used proper parts of the theory of Lie-groups.

In the bulk of the numerical studies I dealt with self-consistent integral-equations, which I solved iteratively. The values of the functions were stored in a grid, whereupon splines were fitted in order to calculate the functions at arbitrary points in the continuum. My codes were written in C and I used the routines of the GNU Scientific Library (GSL). Besides using splines, I also approximated the functions with series expansion in terms of orthogonal Chebyshev polynomials.

My main results are summarized in the following points.

1. A common reformulation of quantum field theories is done with the introduction of auxiliary fields (for example the BRST symmetry of non-abelian gauge theories manifests itself in a very efficient way when auxiliary fields are introduced). These usually refer to composite fields of the original variables. In the recent literature some papers raised doubts on the renormalizability of the O(N) model in this formulation at next-to-leading order (NLO) of the 1/N expansion.

I pointed out, that this statement is false, the theory is renormalizable at NLO also with auxiliary fields. I demonstrated the NLO renormalizability also after eliminating the auxiliary field, and showed the equivalence of the renormalized theories formulated with and without auxiliary field [1].

2. I investigated in details the numerical solution of 2PI renormalized field and propagator equations of the one component ϕ^4 theory, obtained from the 2-loop 2PI effective potential. At zero temperature I developed a method to derive renormalized equations in an explicitly finite form for the *resummed* 1- and 2-point functions, which allowed to avoid the problems of numerical precision induced by the enormous cancellations corresponding to the increasing cutoff. In found the renormalization conditions lead to the same renormalization scheme as defined by the minimal subtraction procedure using counterterms, known from the literature [9]. This allows a direct (numerical) comparison of the numerical solutions of the numerical solutions obtained with explicitly finite equations, or with ones using counterterms [2].

3. Using counterterms known from the literature [9], I solved the field- and propagator-equations numerically. It turned out that, the solution of the regularized equations for large ($\sim$ 10) values of the coupling constant converges unacceptably slowly as the cutoff increases. Obtaining cutoff independent results is therefore requires huge resources from a numerical point of view. I solved this problem by developing a new algorithm, which iteratively develops the counterterms together with the equations. The convergence-improving effect of this method is expected to be a valuable tool for more complicated strongly coupled theories, when one solves self-consistent bare equations numerically [2].

For each method of the numerical studies, I was able to improve the numerical accuracy of the solution of the 2PI equations never achieved in the literature before: the relative error is around 10^{-4} %.

4. Unlike for the O(N) model, the leading large N solution of the U(N)×U(N) symmetric linear matrix model is not known. I found an approximate solution of the field equations in the large-N limit in a broken phase, for a condensate proportional to the unit matrix. The extra approximation assumes the presence of a mass-hierarchy between the scalar and pseudoscalar fields [3]. I have also applied the renormalization programme to this case and obtained explicitly finite field- and propagator-equations. With the numerical study of these equations, I mapped out the region of the parameter space, where the expansion in terms of the pseudoscalar/scalar mass-ratio is valid in a self-consistent way [3].

5. I also derived field equations for a more general diagonal condensate. I managed to show, that with explicit symmetry breaking proportional to the unit matrix, there exists a large region of the parameter space, where the condensate in the "8" direction corresponding to the SU(3) classification shows non-trivial spontaneously broken states [4].

6. I constructed the renormalized effective potential for the solution containing the more general background field. This allowed the determination of the global ground state of the theory. It turned out that one of the non-trivial extrema of the potential found in the 5th point is metastable, but could transform into the global minimum, i.e. the true ground state, when a moderate external source is applied along the "8" direction [4].

Publications of the results referred in the PhD Thesis:

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[3] G. Fejős, A. Patkós, Phys. Rev. D**82**, 045011 (2010)

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